dynamics of broken symmetry

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Non-Equilibrium and String Theory workshop
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Miraculous Joe Bhaseen

far-from equilibrium dynamics

- The challenge: how to characterise **quantum dynamics** far from equilibrium?

- Lack of broadly applicable **principles** and **techniques**

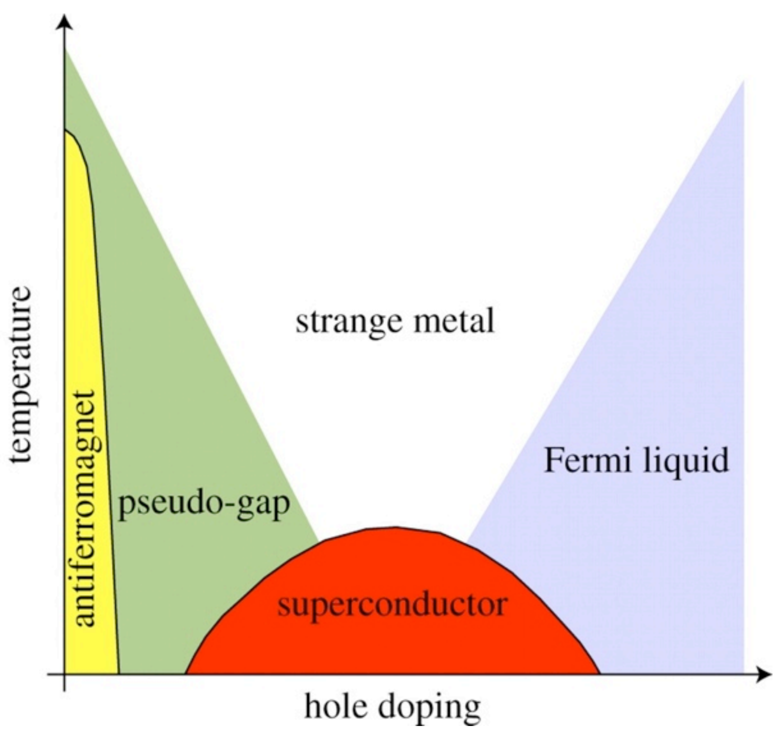
- Progress has been achieved in **integrable** models:
  - quench to CFT: *Cardy & Calabrese* PRL **96** 2006
  - quench in transverse Ising chain: *Calabrese, Essler & Fagottini* PRL **106** (2011)
  - quench in BCS pairing: *Barankov, Levitov & Spivak* PRL **93** (2004)

- **Holography** provides non-integrable yet solvable examples
  - plasma quench: *Chesler & Yaffe* PRL **102** (2009)
  - plasma thermalisation: *de Boer et al*. PRL **106** (2011)
  - AdS/CMT: *Sonner & Green* PRL **109** (2012)

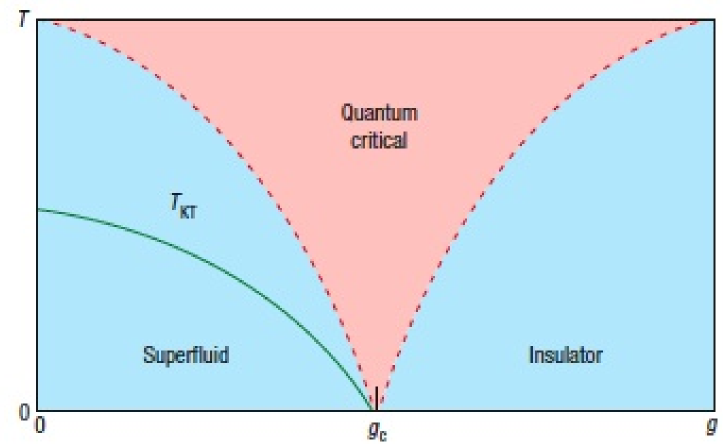
holography in condensed matter

- We will endeavour to model QCPs using holography. **Scaling** symmetries encoded as **isometries** of `dual' spacetime

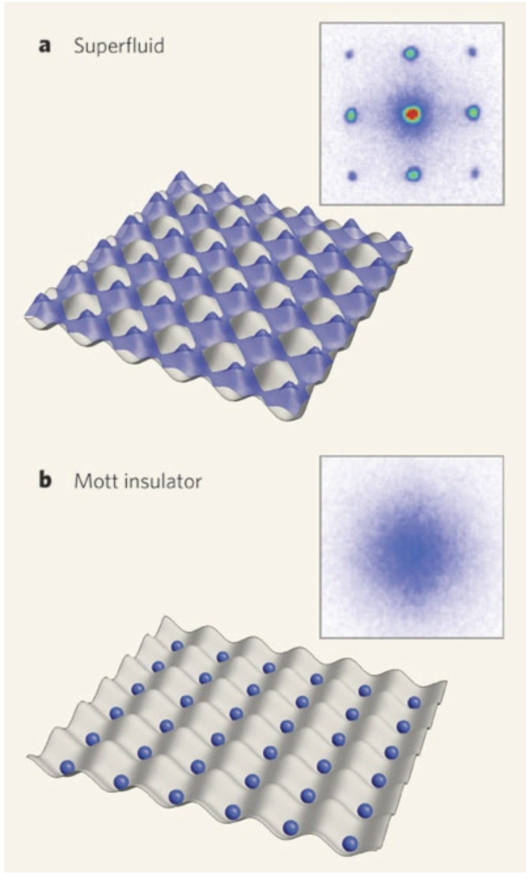
- Continuum theory near QCP is encoded in dynamics of dual string theory



cuprate superconductor



SI transition



realised in cold atoms

outline

1. non-equilibrium background
   “holography provides solvable examples ”

2. model and background
   “holographic superconductors, time-dependent BCS”

3. a holographic setup for dynamical symmetry breaking
   “Numerical relativity, structure of quasi-normal modes”

4. conclusions and outlook
   “generic dynamical consequences of symmetry breaking”

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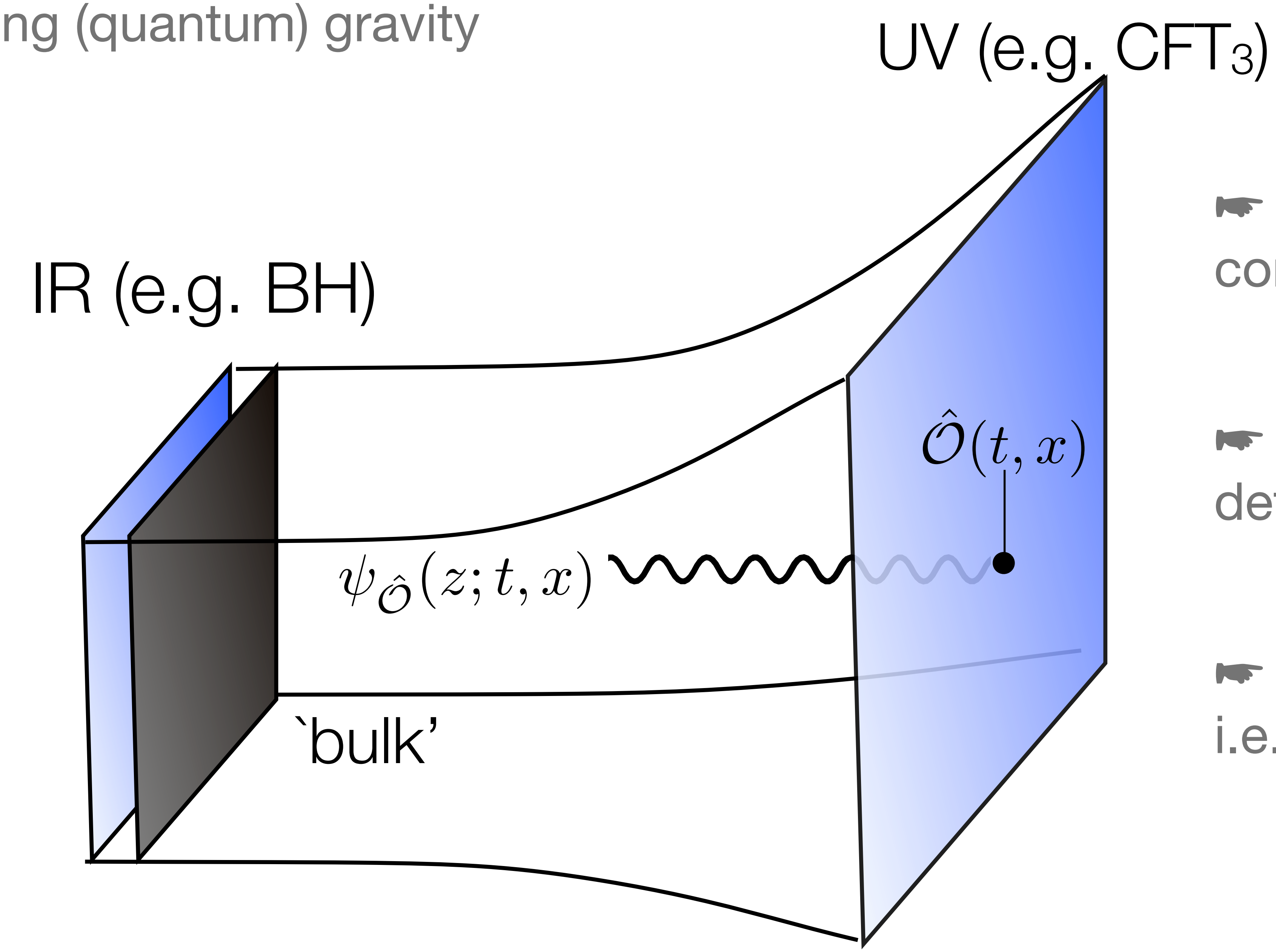
holography in a nutshell

- the effect of **strong correlations** (coupling) is recast into **geometry**: RG flow is incorporated into curved space and correlation functions are computed using (quantum) gravity



☛ set UV boundary conditions

☛ operator insertions/ deformations

☛ usually probe fields, i.e. linear response

a holographic model of superconductivity

- Superconductivity is a manifestation of symmetry breaking. New results here in a dynamical context are **very general** and extend beyond holography

- Specific example: minimal model of holographic superconductor

$$S = \int d^4x \sqrt{-g} \left[R + \frac{6}{\ell^2} - \frac{1}{4} F^2 - |D\psi|^2 - m^2 |\psi|^2 \right]$$

- Complex scalar Ψ is dual to symmetry-breaking order parameter
  1) RN: un-condensed normal phase, new hairy BH: s.c. phase
  [Gubser; Hartnoll, Herzog, Horowitz]
  2) leading near-boundary term of Ψ = source; subleading term = vev
  3) M-theory superconducting solutions exist!
  [Gaunlett, Sonner, Wiseman]

an old dog and a new trick

- BCS theory is the celebrated microscopic explanation of conventional superconductivity. An old story!

BCS hamiltonian: $$\mathcal{H} = \sum_{p,\sigma} \epsilon_p a^\dagger_{p,\sigma} a_{p\sigma} - \frac{\lambda(t)}{2} \sum_{q,p} a^\dagger_{p\uparrow} a^\dagger_{-p\downarrow} a_{-q\downarrow} a_{q\uparrow}$$

BCS groundstate: $$|\Psi(t)\rangle = \prod_p \left[u_p(t) + v_p(t) a^\dagger_{p\uparrow} a^\dagger_{-p\downarrow} \right] |0\rangle$$

pairing gap function: $$\Delta(t) = \lambda \sum_p u_p(t) v_p^*(t)$$

- Recent (2004 -) new developments: the resulting (non-adiabatic) dynamics can be mapped onto a non-linear **integrable** system! [Barankov, Levitov & Spivak; Yuzbashyan, Altshuler, Kuznetsov & Enolskii]

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time-dependent BCS pairing problem

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BL phase diagram

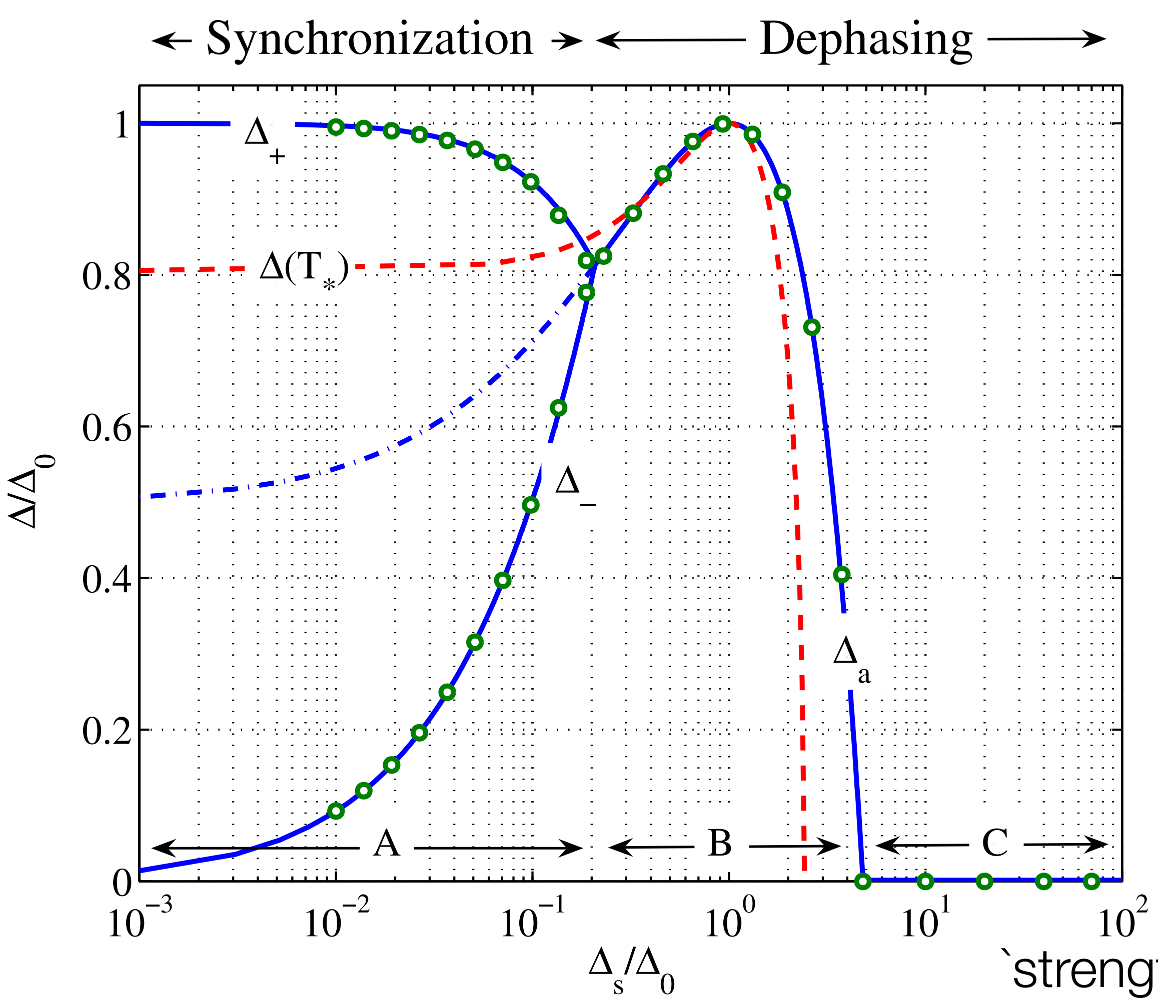
- The dynamics of this quench give rise to **three** distinct regimes

I. Oscillation

II.Decay to finite gap

III.Decay to zero gap

final pairing gap



\`strength' of quench

- our achievement is **twofold**: 1) we exhibit analogous phenomena in a strongly-coupled system, with thermal and collisional damping
  2) we identify a new and **generic** mechanism within dynamical symmetry breaking leading to this behaviour

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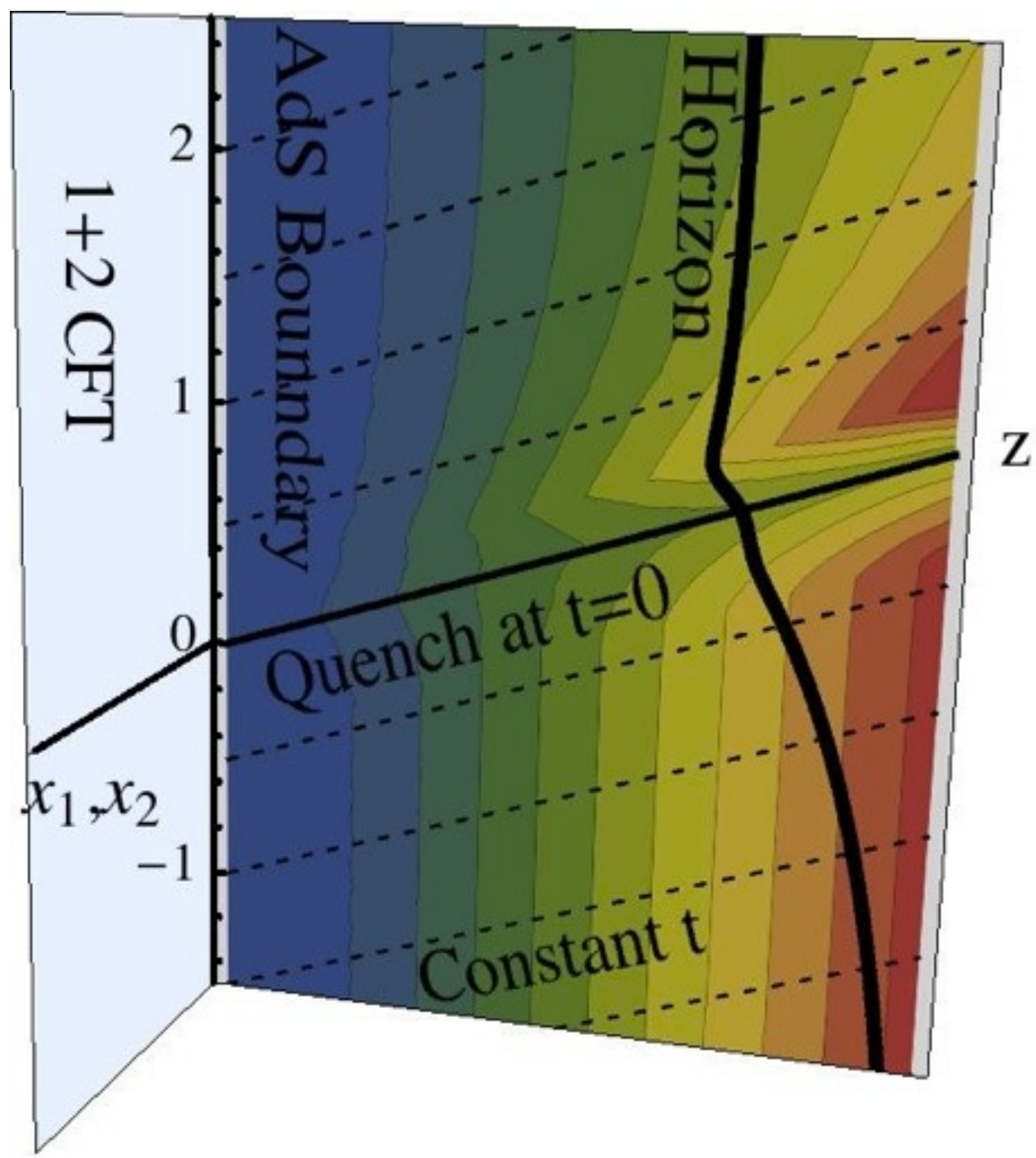
ads/cmt dynamics: numerical relativity

- We wish to model a quench **holographically**: prescribe a sudden change in some physical parameter of the theory on the **boundary** and then evolve the non-linear PDEs numerically to '**fill in the bulk**'



more details of the setup [related work: Murata, Kinoshita & Tanahashi, 2010]

- for simplicity: take **homogeneous** quench

$$ds^2 = \frac{1}{z^2}\left(-T(v,z)\,dv^2 - 2\,dvdz + S(v,z)^2 dx_i^2\right)$$

- the complex scalar can be expressed as

$$\psi(v,z) \quad = \quad z\left(\psi_1(v) + \hat{\psi}(v,z)\right)$$

- and Ψ_1(t) is the **source** at the boundary. Use a spike in the source to quench the system (can think of different systems and different quenches)

- solve system of **(1+1) non-linear PDE** by a pseudo-spectral method in spatial directions and 'Crank-Nicholson' finite differences in time direction (subtle issue about gauges. trial and error leads to stable choice)

the resulting dynamics I

$$\psi_1(t) = \delta e^{-10(t-t_0)^2}$$

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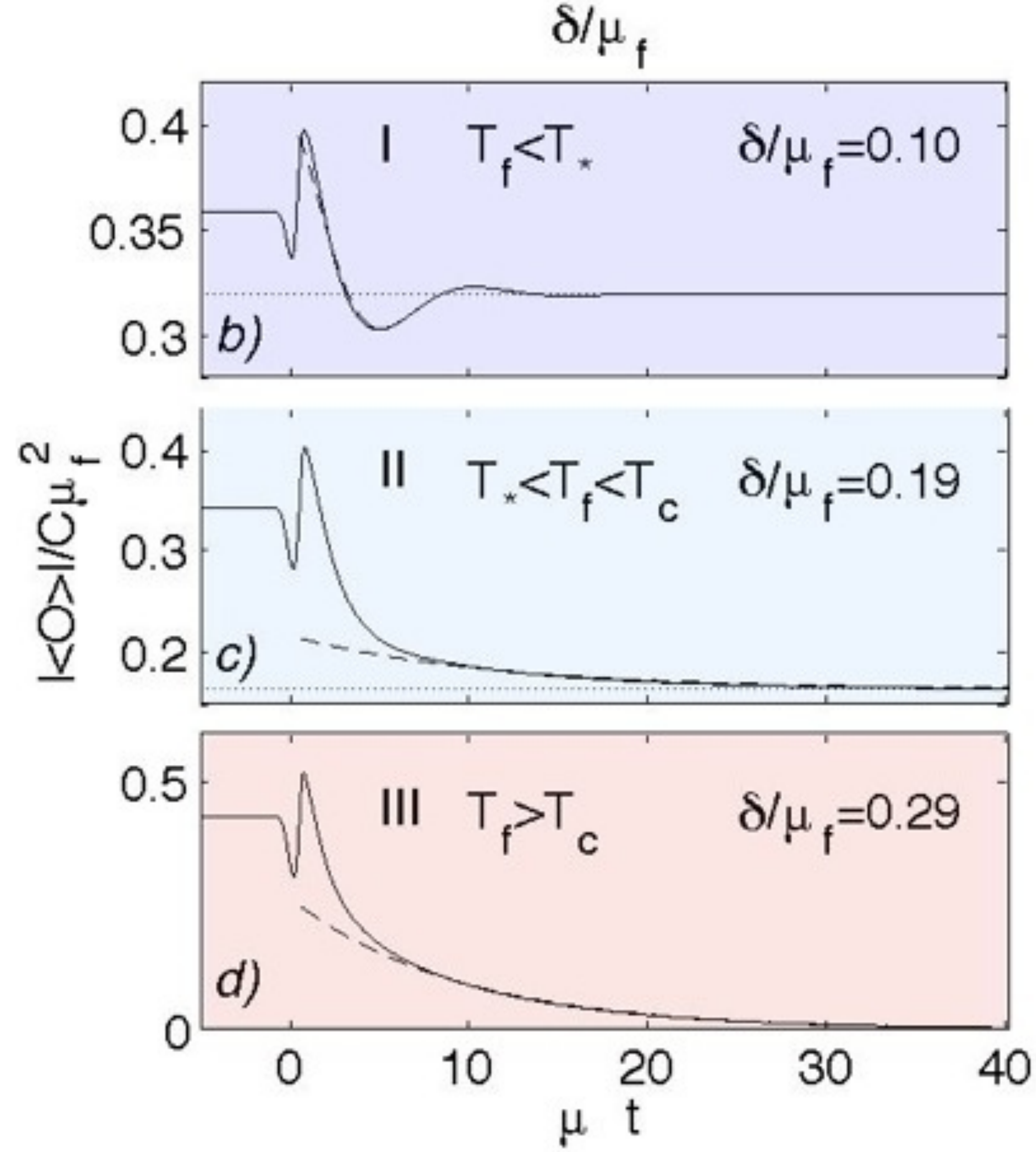
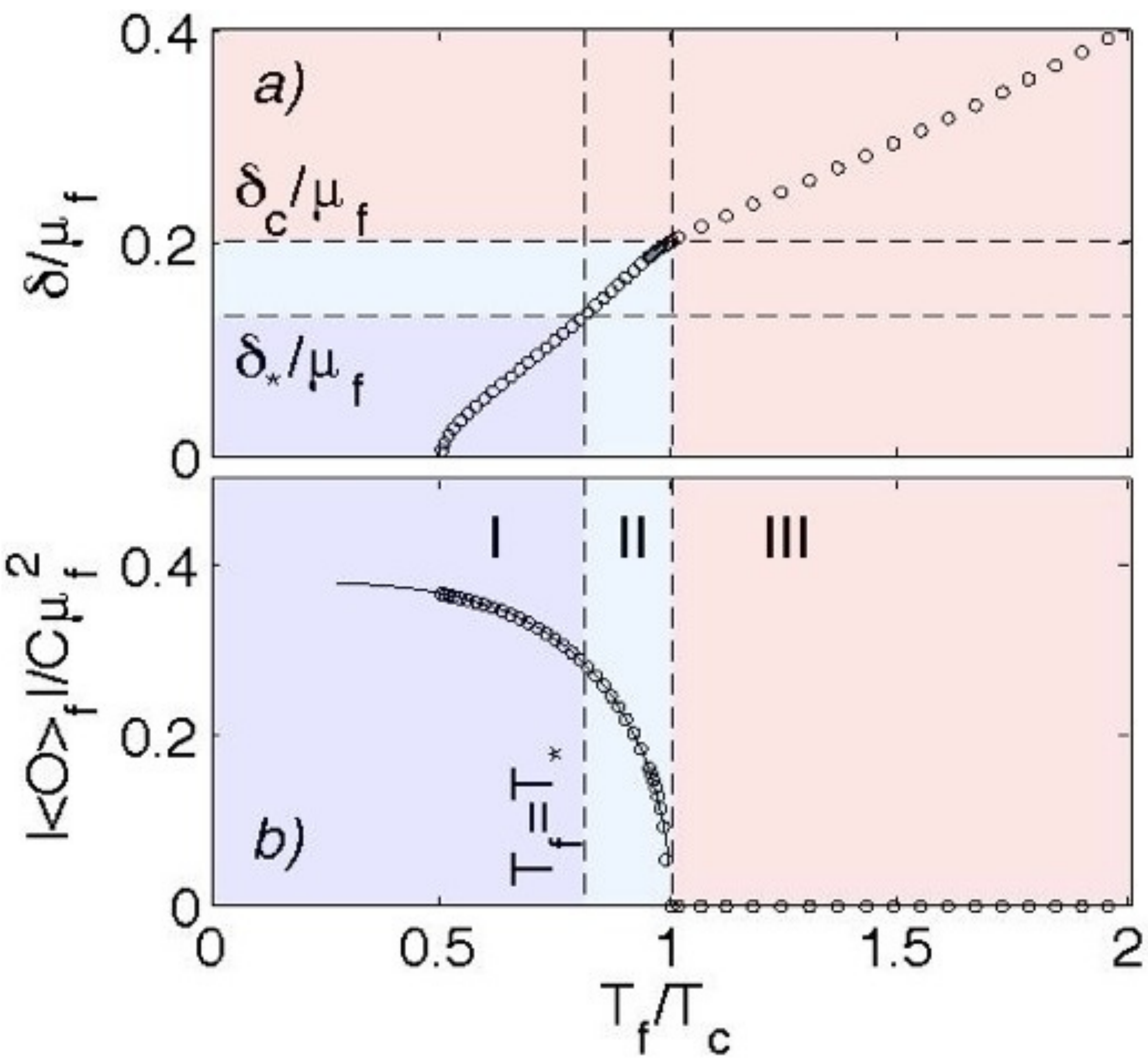


the resulting dynamics II

$$\psi_1(t) = \delta e^{-10(t-t_0)^2}$$

- we can dress the results up as a **dynamical phase diagram**



- BL-type analysis extended to include strong correlations, thermal damping,... we find similar behaviour: great! but why? and how?

clues from quasinormal mode structure

- let us study the structure of **quasi-normal modes** about the final state

$$\begin{aligned} \psi(v,z) &= \psi_0(z) + \delta\psi(v,z) \\ g_{ab}(v,z) &= g_{ab,0}(z) + \delta g_{ab}(v,z) \\ A(v,z) &= A_0(z) + \delta A(v,z) \end{aligned}$$

- deal with diffeo and U(1) gauge symmetry by defining **gauge-invariant variables** (c.f. cosmological perturbation theory)

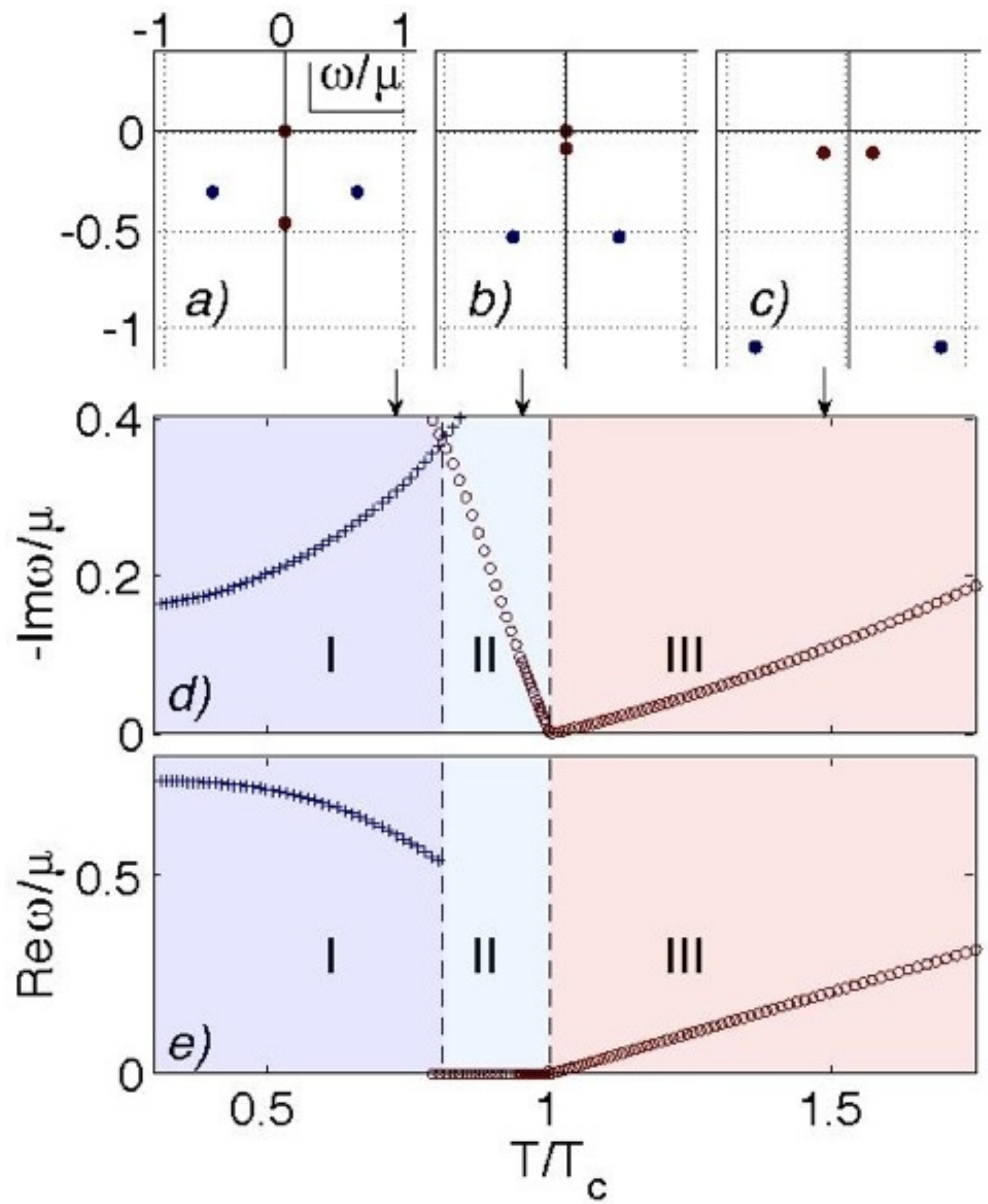
$$\delta\Phi_I(v,z) = e^{-i\omega v}\Phi_I^\omega(z)$$

- The **analytic structure** of the Φ tells us about a) late-time behaviour of observables b) poles in two-point functions of dual operators

quasi-normal mode structure

$$|\langle \mathcal{O}(t) \rangle| = |\langle \mathcal{O}_f \rangle + c e^{-i\omega_L t}|$$

- Off-axis poles lead to **oscillations** in **broken** phase
- Dynamics very well approximated by **leading QNM**
- Very good **quantitative agreement** with non-linear PDE code



quasi-normal pole dance

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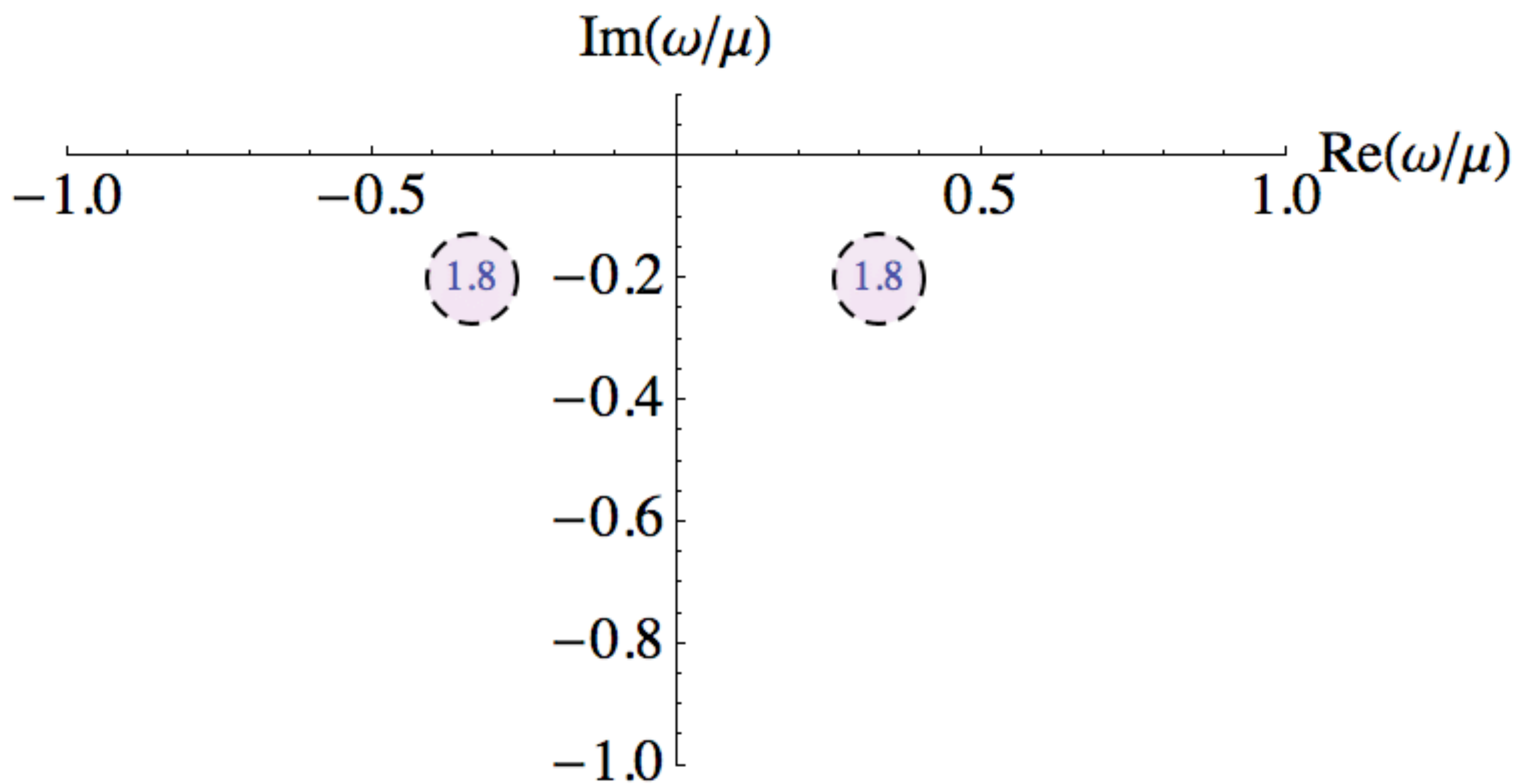

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dynamics of symmetry breaking

- T-reversal invariance means collective mode spectrum (manifested in our example as QNMs) must be **symmetric** under

$$\omega \to -\omega^*$$

- Poles in spectral function (and other observables) come in two varieties:
  a) pairs of poles **off** imaginary axis
  b) single poles **on** imaginary axis

1. S.c. phase transition: coalescence of two poles at TC at $\omega = 0$

2. Broken U(1) ⇒ Single pole (i.e. mode) at $\omega = 0$ (Goldstone mode)

3. At T=0 no source of dissipation ⇒ leading poles are oscillatory in nature

1 + 2 + 3 = BL dynamical phase diagram!

conclusions

- very interesting **far-from-equilibrium** problems are accessible at the intersection of numerical relativity and AdS/CFT.
  speculative comment: exact non-linear PDE methods may well be brought to bear on non-equilibrium field theory!

- simulated a **quantum quench** in ads/cmt: persistence of **BL phenomena** to strong coupling and in systems that thermalise makes it more likely to be observed in actual experiments

- in fact: our analysis shows that BL-type behaviour is **generic** for dynamical breaking of a continuous symmetry. This makes the **experimental point** even more emphatically.

- Are there different contexts? Higgs mechanism, early universe, you name it...
  [e.g. see this week's "the QNM of quantum criticality" by Sachdev]

thanks for your attention!